Technical Notes

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Lifting-Line Solution for a Symmetrical Thin Wing in Ground Effect

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Introduction

DURING landing and takeoff, the aerodynamic characteristics of a wing are influenced by the proximity to the ground. Some of the early attempts at predicting ground effect are outlined in Wetmore and Turner.[1] For incompressible inviscid flow, a lifting-surface solution is given by Saunders[2] and a vortex-lattice solution by Kalman, Rodden, and Giesing.[3] Currently, numerical solutions can be obtained with the use of panel methods, and the capability exists to compute the location of the trailing vortex sheet.

It is the purpose of this Note to present a lifting-line solution to the problem that is easier to obtain than the numerical solutions referred to above but that provides for reasonable accuracy for the larger-aspect-ratio cases. Also, the effect of thickness on lift will be considered in both the lifting-line and vortex-lattice formulations.

Method of Solution

Consider the steady, inviscid incompressible flow of a uniform stream of speed V_∞ at angle of attack α past a thin rectangular wing of span b and chord c (aspect ratio $A=b/c$), whose quarter-chord line is at a distance h above the ground. y represents the distance along the lifting line from midspan, and x is in the stream direction. The wing surface is

$$z=-\alpha x\pm\epsilon f(x,y) \tag{1}$$

where ϵ is the thickness ratio.

Consider the two-dimensional aerodynamics (including ground effect) at a typical spanwise station. For small α and ϵ, the sectional lift coefficient can be written as

$$C_\ell=a_0\alpha+a_\epsilon\epsilon=2\Gamma/V_\infty c \tag{2}$$

where Γ is the circulation. For large values of h/c, the thin-airfoil solution of Plotkin and Kennell[4] can be used for the aerodynamic coefficients. The angle-of-attack (flat-plate) result is

$$a_0=2\pi\left[1+\frac{1}{16}\left(\frac{c}{h}\right)^2-\frac{3}{512}\left(\frac{c}{h}\right)^4+0\left(\frac{c}{h}\right)^6\right] \tag{3}$$

For a Joukowski-like airfoil with thickness function

$$\frac{2c}{3^{3/2}}\epsilon\left(1-\frac{2x}{c}\right)\left(1-\frac{4x^2}{c^2}\right)^{1/2}$$

we get

$$a_\epsilon=\frac{4}{3^{3/2}}\left[-\frac{3}{128}\left(\frac{c}{h}\right)^3+\frac{5}{2048}\left(\frac{c}{h}\right)^5+0\left(\frac{c}{h}\right)^7\right] \tag{4}$$

For a large-aspect-ratio wing, the lifting-line equation is (see Karamcheti[5])

$$\Gamma=0.5V_\infty c[a_0(\alpha-\alpha_i)+a_\epsilon\epsilon] \tag{5}$$

where a_0 and a_ϵ are given in Eqs. (3) and (4). In the absence of the ground plane, the lifting solution is modeled by the lifting line and its trailing vortex sheet, and the thickness solution is modeled by a distribution of sources on the wing midplane of strength $\epsilon f_x(x,y)$ per area. The induced angle of attack α_i for the present problem is due to the trailing vortex sheet, its image in the ground plane, and the image of the source distribution.

The integral equation for Γ is obtained from Eq. (5) by expressing the induced angle of attack in terms of the unknown vortex and given source distributions as noted above. This equation is then solved using the method of Glauert (see Ref. 5) as follows. Let

$$\Gamma=2bV_\infty\sum_1^N A_n\sin n\theta,\quad \theta=\cos^{-1}\left(\frac{2y}{b}\right) \tag{6}$$

Substitution of Eq. (6) into the integral equation for Γ yields an equation for the A_n (note that G and $\alpha_{i\epsilon}$ represent the contribution to α_i from the images of the trailing vortex sheet and the wing source distribution, respectively).

$$\sum_1^N A_n\sin n\theta\left\{\sin\theta+\frac{a_0 n}{4\pi A}(\pi+\sin\theta G(\theta)\right\}$$
$$=\frac{a_0\sin\theta}{4A}\left\{\alpha-\alpha_{i\epsilon}+\frac{a_\epsilon\epsilon}{a_0}\right\} \tag{7}$$

where

$$G=\int_0^\pi d\theta_0\cos n\theta_0\left\{\frac{\cos\theta-\cos\theta_0}{(\cos\theta-\cos\theta_0)^2+16h^2/b^2}\right\} \tag{8}$$

and

$$\alpha_{i\epsilon}=-\frac{2h\epsilon}{\pi bA}\int_0^\pi d\theta_0\int_{-1}^1 d\xi\, f_x[\xi,y(\theta_0)]$$
$$\times\frac{\sin\theta_0}{\{(\xi/A)^2+(\cos\theta-\cos\theta_0)^2+16h^2/b^2\}^{3/2}} \tag{9}$$

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Equation (7) is solved by collocation using values of G and α_{i_ϵ} obtained by numerical integration. Once the A_n are determined, the wing lift coefficient can be obtained in the form

$$C_L = C_{L_\alpha}\alpha + C_{L_\epsilon}\epsilon \tag{10}$$

Results and Discussion

The lifting-surface[2] and vortex-lattice[3] solutions are given for rectangular wings with $A=1$, 2, and 4 and agree well with each other and experiments. For comparison purposes, a vortex-lattice method was developed. For $A=4$, these vortex-lattice results agree well with those in Refs. 2 and 3 and will be used in the figures. To handle the problem with thickness, the downwash at the control points included the contribution from the distributed source images obtained by numerical integration.

The slope of the wing lift coefficient vs angle-of-attack curve C_{L_α} is plotted vs $(2h/b)$ in Figs. 1 and 2 for $A=4$ and 6, respectively. $(C_{L_\alpha})_\infty$ is the result for $h\to\infty$. The lifting-line results are seen to agree well with the vortex-lattice results, with better agreement at the larger aspect ratio as expected. The lifting-line results can be extended to smaller values of $(2h/b)$ if the exact two-dimensional value of a_0 determined by Havelock[6] is used.

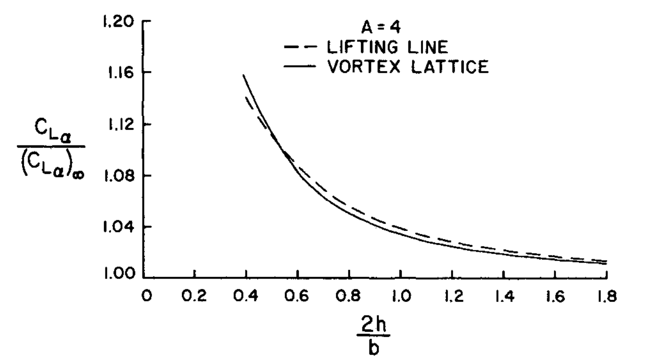

Fig. 1 Lift curve slope for flat rectangular wing of aspect ratio 4.

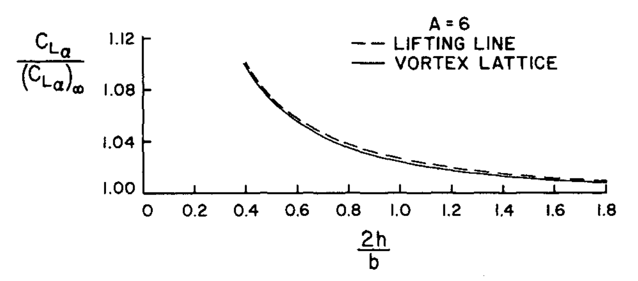

Fig. 2 Lift curve slope for flat rectangular wing of aspect ratio 6.

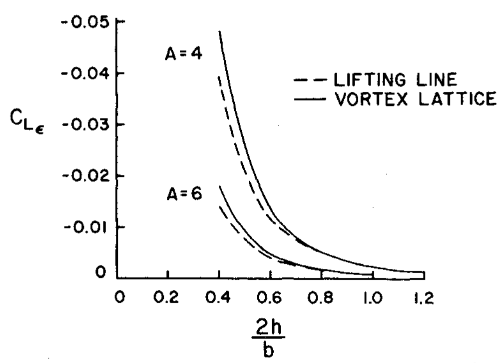

Fig. 3 Slope of lift coefficient vs thickness ratio curve for flat rectangular wings of aspect ratios 4 and 6.

The slope of the wing lift coefficient vs thickness ratio curve C_{L_ϵ} is plotted vs $(2h/b)$ in Fig. 3. The results of the lifting-line and vortex-lattice calculations again are seen to agree reasonably well, and thickness is seen to decrease the lift.

As a final point of interest, the lifting-surface, vortex-lattice, and lifting-line theories discussed here all assume a linear relationship between lift and angle of attack. For the two-dimensional, zero-thickness ground effect problem, an expansion of the lift coefficient of Havelock[6] in both c/h and α, keeping terms to $0(\alpha^2)$, yields

$$C_\ell = 2\pi\alpha\left\{1-\frac{\alpha}{2}\left(\frac{c}{h}\right)+\frac{1}{16}\left(\frac{c}{h}\right)^2+\dots\right\} \tag{11}$$

For smaller values of h/c, it is seen that the presence of the ground introduces a nonlinearity in α much stronger than in the infinite fluid problem. It is therefore expected that the wing in ground effect results of Refs. 2 and 3 and those presented here are valid for a more restricted range of angle of attack than is usually appropriate for these theories in the absence of the gound plane.

Acknowledgment

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Integration of Singular Functions Associated with Lifting Surface Theory

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Nomenclature

C,c	= constants
E	= error term in integration rules
F	= function
f,g	= arbitrary functions
H	= quadrature weights
I	= integral
i_n	= index from 1 to n

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